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Solution by the PROPOSER.

$x^3+y^3=20^3\times105489$ or $=843912000$. Take $x=1$, 2, etc., until we find $x=15$, $y=945$. Put $945=a$ and $15=b$. Then since there must be four values for each, we have, $x^3+y^3=a^3+b^3$, or $x^3-b^3=a^3-y^3$ ....(1).

Now suppose $x^3+b^3=a^3-y^3$. Let $x=a-u$ and $b=mu-y$.

$\therefore a^3-3a^2u+3au^2-u^3+m^3u^3-3m^3a^2y+3muy^2-y^3=a^3-y^3$.

Let $m=a^2/y^2$ and we have $u=3ay^3/(a^3+y^3)$.

$\therefore x=[a(a^3-2y^3)]/(a^3+y^3)$ and $b=[y(2a^3-y^3)]/(a^3+y^3)$.

Now to find x and y in $x^3-b^3=a^3-y^3$ it is evident one of the above values must be taken negatively, but it cannot be the value of b since the result would be at least one negative value.

$\therefore$ in (1) we have, $x=[y(2a^3-y^3)]/(a^3+y^3)$ and $b=[a(2y^3-a^3)]/(a^3+y^3)$. Whence we find $y=a\sqrt[3]{[(a+b)/(2a-b)]}=945\sqrt[3]{(\frac{960}{1875})}=945\sqrt[3]{(\frac{64}{125})}$.

$\therefore y=756$, and we find $x=744$.

$\therefore x=15$, 744, 756, and 945 ; $y=945$, 756, 744, and 15.

Also solved by *J. H. DRUMMOND*.

74. Proposed by SYLVESTER ROBINS, North Branch Depot, N. J.

It is required to take from the proper *key* suitable material and hastily construct a "nest" of 10 or 15 prime, integral, rational trapeziums, each containing an area equal to the square root of the product of its four sides.

Solution by the PROPOSER.

If the business require great haste, write n, —two or more, convergents in the expansion of any number of the form of $\sqrt{(a^2+1)}$, say $\sqrt{2}$, $\sqrt{5}$, $\sqrt{10}$, $\sqrt{17}$, $\sqrt{26}$, $\sqrt{37}$, etc. Observe the number of trapeziums, $[n(n-1)]/2$ is always triangular.

$\sqrt{17}=\frac{4}{1}$, $\frac{33}{8}$: $4^2=\mathbf{16}$, $17\times1^2=\mathbf{17}$, $17\times8^2=\mathbf{1088}$, $33^2=\mathbf{1089}$.

$\sqrt{10}=\frac{3}{1}$, $\frac{19}{6}$, $\frac{117}{37}$: $3^2=9$, $10\times1^2=10$, $19^2=361$, $10\times6^2=360$, $117^2=13689$, $10\times37^2=13690$. 9, 10, 360, 361 ; 9, 10, 13689, 13690 ; 360, 361, 13689, 13690.

$\sqrt{5}=\frac{2}{1}$, $\frac{9}{4}$, $\frac{38}{17}$, $\frac{161}{72}$: 4, 5, 80, 81 ; 4, 5, 1444, 1445 ; 4, 5, 25920, 25921 ; 80, 81, 1444, 1445 ; 80, 81, 25920, 25921 ; 1444, 1445, 25920, 25921.

$\sqrt{2}=\frac{1}{1}$, $\frac{3}{2}$, $\frac{7}{5}$, $\frac{17}{12}$, $\frac{41}{29}$: 1, 2, 8, 9 ; 1, 2, 49, 50 ; 1, 2, 288, 289 ; 1, 2, 1681, 1682 ; 8, 9, 49, 50 ; 8, 9, 288, 289 ; 8, 9, 1681, 1682 ; 49, 50, 288, 289 ; 49, 50, 1681, 1682 ; 288, 289, 1681, 1682.

Should it be desired to obtain several nests of this kind of rational trapeziums from a single series, take the convergents from the expansion of quantities of the form $\sqrt{(a^2+b)}$, where b is greater than 1.

$\sqrt{3}=\frac{1}{1}$, $\mathbf{2/1}$, $\frac{5}{3}$, $\mathbf{7/7}$, $\frac{19}{11}$, $\mathbf{26/15}$. $\frac{71}{41}$, $\mathbf{97/56}$, $\frac{265}{153}$: 3, 4, 48, 49 ; 3, 4, 675, 676 ; 3, 4, 9408, 9409 ; 48, 49, 675, 676 ; 48, 49, 9408, 9409 ; 675, 676, 9408, 9409 ; .....

1, 3, 25, 27 ; 1, 3, 361, 363 ; 1, 3, 5041, 5043 ; 1, 3, 70225, 70227 ; 27, 25, 361, 363 ; 27, 25, 5041, 5043 ; 27, 25, 70225, 70227 ; 361, 363, 5041, 5043 ; 361, 363, 70225, 70227 ; 5041, 5043, 70225, 70227.

$\surd6=\frac{2}{1}\ldots\ldots\frac{22}{9}\ldots\ldots\frac{218}{89}\ldots\ldots\frac{2158}{881}.\ \ldots\ldots\frac{21362}{8721}\ldots\ldots\frac{211462}{86329}.$

Here is material for 15 trapeziums of the kind required.

$\surd6=\frac{5}{2}\ldots\ldots\frac{49}{20}\ldots\ldots\frac{485}{198}\ldots\ldots\frac{4801}{1960}\ldots\ldots\frac{47525}{19402}\ldots\ldots\frac{470449}{192060}\ldots\ldots\frac{4656965}{1901198}.$

These convergents furnish material for 21 trapeziums.

$\surd7=\frac{2}{1}, \frac{3}{1}, \frac{5}{2}, \frac{8}{3}, \frac{37}{14}, \frac{45}{17}, \frac{82}{31}, \frac{127}{48}, \frac{590}{223}, \frac{717}{271}, \frac{1307}{494}, \frac{2024}{765}, \frac{9403}{3554}, \frac{11427}{4319}, \frac{20830}{7873}, \frac{32257}{12192}, \frac{149858}{56641}, \frac{182115}{68833}$. In the nine odd convergents may be found the roots of 36 trapeziums. In the 2d, 6th, 10th, 14th, 18th, the roots of 10 trapeziums, and in the 4th, 8th, 12th, 16th, the seed-corn of 6 trapeziums of the kind desired.

Also solved by *G. B. M. ZERR.*

NOTE. Professor Zerr's solution of No. 60 is splendid, but he has left out a very difficult part of the solution, and has not referred us to the solution elsewhere. h, k, l, m, n, and p must be so taken that the sum of their fifth powers is a perfect fifth power, and I do not remember where I can find a solution of that question. *J. H. Drummond.*

MISCELLANEOUS.

59. II. Solution by C. W. M. BLACK, A. M., Professor of Mathematics, Wesleyan Academy, Wilbraham, Mass.

[See problem and solution I, in May number, page 156.] Consider the rays of light in a plane parallel to the axis of cylinder. It is easily seen that they are all reflected from an element of the cylinder, parallel and in another plane which together with the given plane makes equal angles with the normal plane along the reflecting element. These planes of reflection, being all perpendicular to the base, will intersect in lines perpendicular to the base, and the locus of the ultimate intersections of adjacent planes will be a caustic cylinder whose elements are perpendicular to the base of the given cylinder. As the direction of the rays of light is still oblique the intersection of the caustic cylinder with the base will be a luminous curve. The elevation of the sun does not affect the positions of the planes of light, and so does not affect the form of the caustic cylinder or of the luminous curve.

The curve is the envelope of the projection upon the base of the successive planes of light. Let circle represent base of cylinder, AB projection of incident plane, BC of reflected plane, AB being parallel to axis of X (see figure in May number, page 156.)

$OB=r$, $\angle ABO=\angle CBO=\angle BOK=\alpha$, $BKX=2\alpha$. Equation to BK is $y=(x-OK)\tan2\alpha=(x-\frac{1}{2}r\sec\alpha)\tan2\alpha$.

Let $\tan\alpha=a$. Then $y=[x-\frac{1}{2}r\surd(1+a^2)][(2a)/(1-a^2)]$............(1).

Differentiating with reference to a, equating to zero, and clearing of fractions, we have

$$(a^2-1)a^2r+2x(1+a^2)^{\frac{3}{2}}-r(1+a^2)^2=0\,;$$

whence $x/r=(1+3a^2)/[2(1+a^2)^{\frac{3}{2}}]$. Substituting in (1),

$$y/r=\{(1+3a^2)/[2(1+a^2)^{\frac{3}{2}}]-[(1+a^2)^{\frac{1}{2}}/2]\}[(2a)/(1-a^2)]=a^3/(1+a^2)^{\frac{3}{2}}.$$

$$\therefore\ (y/r)^{\frac{1}{3}}=a/(1+a^2)^{\frac{1}{2}}\,;\ (y/r)^{\frac{1}{3}}=a^2/(1+a^2).$$

$a^2=(y/r)^{\frac{2}{3}}/[1-(y/r)^{\frac{2}{3}}]$; $\surd(1+a^2)=1/\surd[1-(y/r)^{\frac{2}{3}}]$ ....(2).